Pairwise interaction:

where and are the position vectors of micro-disks;

is the vector pointing from the centre of micro-disk *j* to the centre of micro-disk *i*;

and are the orientations of micro-disks;

*d* is the edge-edge distance;

is the angle of dipole moment with respect to . It is assumed to be the same for both micro-disks, as in scheme S1 in the section on magnetic dipole force and torque calculation.

is the instantaneous spin speed of micro-disks;

is the orientation of the magnetic field;

is the rotation speed of the magnetic field;

is the radius of micro-disk (150 µm);

is the dynamic viscosity of water (10-3 Pa·s);

is the density of water (103 kg/m3);

is the magnetic dipole moment of the micro-disks (10-8 A·m2);

is the magnetic field strength (10 mT);

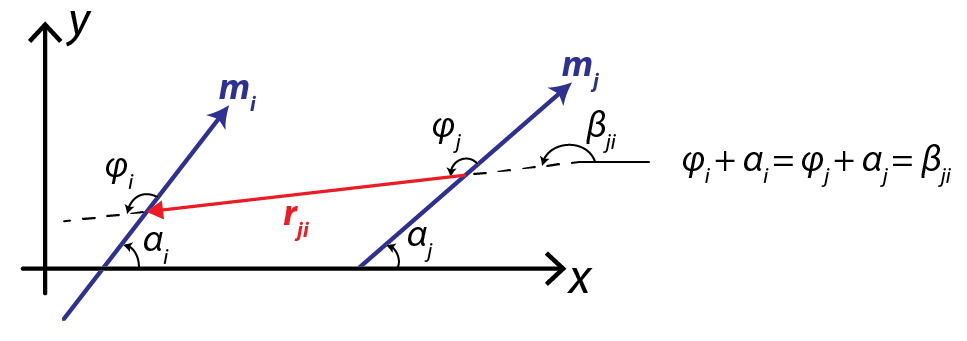
*C* is an adjustable coefficient.

and are the magnetic dipole force on and off the centre-to-centre axis, respectively, and they are functions of and ; (See the section on magnetic dipole force and torque calculation for details)

is the magnetic dipole torque, and it is a function of and .

**Magnetic dipole force and torque calculation**

The geometry of interaction between two magnetic dipoles is shown in scheme 1 below.



Scheme S1. The geometry of dipole-dipole interaction. The blue arrows are the directions of the magnetic dipole of micro-disks. The red arrow is the centre-to-centre axis of micro-disks.

The force by dipole *j* on dipole *i* (K. W. Yung, P. B. Landecker, D. D. Villani, An Analytic Solution for the Force Between Two Magnetic Dipoles. *Magn. Electr.* Sep. 9, 39–52 (1998)):

|  |  |
| --- | --- |
|  | () |
|  | () |

where

the hat denotes a unitized vector;

is the vector pointing from disk *j* to disk *i*;

is the vacuum permeability;

*mi* and *mj* are the magnetic moments of micro-disks.

*αi* and *αj* are defined in Scheme 1.

With the geometric relations

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the force equation becomes

|  |  |  |
| --- | --- | --- |
|  |  | () |

Set , then

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |
|  |  | () |

The torque by dipole *j* on dipole *i* (P. B. Landecker, D. D. Villani, K. W. Yung, Analytic solution for the torque between two magnetic dipoles. *Magn. Electr. Sep.* 10, 29–33 (1999)):

|  |  |  |
| --- | --- | --- |
|  |  | () |
|  |  | () |

Set , then

|  |  |  |
| --- | --- | --- |
|  |  | () |